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In this letter we argue that the event-by-event fluctuations of the ratio of the positively charged and the negatively charged pions provides a signal of quark-gluon plasma. The fact that quarks carry fractional charges is ultimately responsible for this distinct signal.

It is of great importance that we have a clear signal of the long-sought quark-gluon plasma (QGP) not only for the experiments at RHIC but also for theoretical reasons. At stake is our fundamental understanding of strong interactions as well as understanding of the state of matter in the very early universe [1]. Proposed signals of this new state of matter abound in literature [2] one of the most studied being the J/ψ suppression [3,4]

In this paper, we propose the event-by-event h^+/h^- fluctuations as a "smoking gun" signal of QGP. We would also like to stress that this observable is something that has already been calculated on a lattice. The idea is very simple and is reminiscent of the original detection of color in e^+e^- experiment where one measures

$$R_{e^+e^-} \equiv \frac{e^+e^- \to \text{ Hadrons}}{e^+e^- \to \mu^+\mu^-} = N_c \sum_q Q_q^2 \tag{1}$$

Here Q_q is the charge of each flavor and N_c is the number of colors. Note that if the fundamental degrees of freedom were hadrons, then $R_{e^+e^-}$ would be far different than this simple counting. We would like to establish that the event-by-event h^+/h^- fluctuations can similarly determine whether the underlying degrees of freedom are quarks and gluons or hadrons.

The point is that in QGP phase, the unit of charge is 1/3 while in the hadronic phase, the unit of charge is 1. The net charge, of course does not depend on such subtleties, but the fluctuation in the net charge depends on the *squares* of the charges and hence is strongly dependent on which phase it originates from. However, measuring the charge fluctuation itself is plagued by systematic uncertainties such as volume fluctuations. In a previous letter [5], we showed that the ratio fluctuation is only sensitive to the *density* fluctuations but not to volume fluctuations and hence provides a much cleaner signal. The task for us is then to find a suitable ratio whose fluctuation is easy to measure and simply related to the net charge fluctuation.

The obvious candidate is the ratio $F = Q/N_{\rm ch}$ where

$$Q = N_+ - N_- \tag{2}$$

is the net charge and

$$N_{\rm ch} = N_+ + N_- \tag{3}$$

is the charge multiplicity. Here $N_\pm$ denote the charge multiplicities. Instead of using F, however, in this paper we propose to use the charge ratio $R = N_+/N_-$. The advantages of using R over F are that although trivially related, R is more fundamental to experiments and the signal is about 4 times amplified in R as we show below.

To relate R with F, we first rewrite the charge ratio as

$$R = \frac{N_+}{N_-} = \frac{1+F}{1-F} \tag{4}$$

When $\langle N_{\rm ch}\rangle \gg \langle Q\rangle$ we can safely say $|F| \ll 1$. Expanding in terms of F yields

$$R \approx 1 + 2F + 2F^2 \tag{5}$$

Defining $\delta x = x - \langle x\rangle$ for any fluctuating quantity x, it is easy to show

$$\left\langle \delta R^2 \right\rangle = \left\langle R^2 \right\rangle - \left\langle R \right\rangle^2 \approx 4 \left\langle \delta F^2 \right\rangle \tag{6}$$

where $\langle \cdots \rangle$ denotes the average over all events.

Let us now consider $\left\langle \delta F^2 \right\rangle$ more closely. In a previous letter [5] (see also [6] and the upcoming paper [7]), we showed that a ratio fluctuation can be expressed as

$$\left\langle \delta F^2 \right\rangle = \frac{\langle Q\rangle^2}{\langle N_{\rm ch}\rangle^2} \left\langle \left(\frac{\delta Q}{\langle Q\rangle} - \frac{\delta N_{\rm ch}}{\langle N\rangle_{\rm ch}} \right)^2 \right\rangle \tag{7}$$

We then showed that when the average ratio is very much different than 1, then the fluctuation is driven mainly by the fluctuation in the smaller quantity (for instance K/π fluctuation is driven by K fluctuation). At RHIC we expect $\langle Q\rangle / \langle N_{\rm ch}\rangle \sim 5\%$. Hence the fluctuation in F is totally dominated by the fluctuation in Q so that

$$\left\langle \delta F^2 \right\rangle \approx \frac{\left\langle \delta Q^2 \right\rangle}{\langle N_{\rm ch}\rangle^2} \tag{8}$$

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If we can detect all charged particles from a heavy ion collision, the net charge Q is a fixed quantity and hence will not fluctuate. This implies that $\langle \delta F^2 \rangle$ is very small with a 4π coverage. However, no detector can catch all charged particles. Our study [5,7] shows that for a realistic detector acceptance, using the grand canonical ensemble is acceptable and that is what we assume here. Our main observable is then,

$$\langle N_{\rm ch} \rangle \langle \delta R^2 \rangle = 4 \langle N_{\rm ch} \rangle \langle \delta F^2 \rangle = 4 \frac{\langle \delta Q^2 \rangle}{\langle N_{\rm ch} \rangle} \tag{9}$$

to the leading order in the fluctuations and $1/\langle N_{\rm ch} \rangle$.

So far, we have only considered statistics of the ratio fluctuations. Physics lies in how the charge fluctuation is expressed in terms of the fluctuations in the fundamental degrees of freedom. For simplicity, let us consider a pion gas and a QGP consisting of u and d quarks and gluons. Our main conclusion does not depend on this simplifying assumption. We will briefly consider the size of the corrections towards the end of the paper.

In a pion gas, the fundamental degrees of freedom are of course pions. Hence, $N_{\rm ch} = N_{\pi^+} + N_{\pi^-}$ and

$$\delta Q = \delta N_{\pi^+} - \delta N_{\pi^-} \tag{10}$$

Using thermal distributions and disregarding correlations, we get

$$\langle \delta Q^2 \rangle = \langle \delta N_+^2 \rangle + \langle \delta N_-^2 \rangle = w_\pi \langle N_{\rm ch} \rangle \tag{11}$$

where

$$w_\pi \equiv \langle \delta N_\pi^2 \rangle / \langle N_\pi \rangle \tag{12}$$

is slightly bigger than 1 [5,6]. Hence for a pion gas,

$$\langle N_{\rm ch} \rangle \langle \delta R^2 \rangle \approx 4 \,. \tag{13}$$

For a quark-gluon plasma,

$$\delta Q = Q_u \delta N_u + Q_d \delta N_d \tag{14}$$

where $Q_{u,d}$ and $N_{u,d}$ are the charges of the quarks and the number of the quarks. The fluctuations $\left\langle \delta N_{u,d}^2 \right\rangle$ are measured on lattice [9] and we will shortly get back to the results. For now, let us consider a thermalized gas of non-interacting quarks and gluons to get a physical baseline to compare with. Thermal distributions and no correlations between u and d quarks yield

$$\langle \delta Q^2 \rangle = Q_u^2 w_u \langle N_u \rangle + Q_d^2 w_d \langle N_d \rangle \tag{15}$$

where

$$w_q \equiv \langle \delta N_q^2 \rangle / \langle N_q \rangle \tag{16}$$

is slightly smaller than 1 due to the fermionic nature of quarks.

Relating the final charged particle multiplicity $N_{\rm ch}$ to the number of primordial quarks and gluons is not as simple. To make an estimate, we assume that the entropy is conserved [8] and that all the particles involved are massless, in thermal equilibrium and non-interacting. For such particles, the following relation between the entropy density and the particle number density holds:

$$\sigma_B = 3.6 n_B \tag{17}$$

and

$$\sigma_F = 4.2 n_F \tag{18}$$

where the subscript B, F signifies the particle types. The total entropy of a quark-gluon gas in a give volume V_{qg} is then

$$S = V_{qg} \sigma_{qg} = 3.6 N_g + 4.2 N_q \tag{19}$$

where $N_{q,g}$ are the number of quarks and gluons inside the volume. As the volume expands and cools, eventually the quarks and gluons are converted to pions. Since entropy is conserved, the pions coming from these quarks and gluons must be given by

$$N_\pi = \frac{S}{3.6} = N_g + \frac{4.2}{3.6} N_q \tag{20}$$

The charged multiplicity is 2/3 of N_π due to isospin symmetry:

$$N_{\rm ch} = \frac{2}{3} \left(N_g + 1.2 N_u + 1.2 N_d \right) \tag{21}$$

Then for massless non-interacting quarks and gluons,

$$\frac{\langle \delta Q^2 \rangle}{\langle N_{\rm ch} \rangle} = \frac{Q_u^2 w_u \langle N_u \rangle + Q_d^2 w_d \langle N_d \rangle}{\frac{2}{3} \left(\langle N_g \rangle + 1.2 \langle N_u \rangle + 1.2 \langle N_d \rangle \right)} = 0.19 \tag{22}$$

Equivalently, our observable has the value

$$\langle N_{\rm ch} \rangle \langle \delta R^2 \rangle \approx 3/4 \tag{23}$$

which is more than a factor of 5 smaller than the pion gas result! This is an unmistakable signal of QGP formation from 'Day-1' measurements.

We now would like to stress that $\langle \delta Q^2 \rangle / \langle N_{\rm ch} \rangle$ is already calculated on lattice and hence one does not have to rely on the above thermal model calculation. In Ref. [9], Gottlieb et. al report their calculation of the quark number susceptibility and the entropy density with 2 flavors of dynamic quarks. These two quantities are directly related to the net charge fluctuation and the charged multiplicity in the following way.

From the definition of the charge susceptibility χ_q, it is clear that

$$\langle \delta Q^2 \rangle = V_{qg} T \chi_q \tag{24}$$

where V_{qg} is the volume of the quark-gluon plasma at the hadronization and T is the temperature. Gottlieb et. al calculated the quark number density susceptibility

$$\chi_S = V_{qg}\beta \left\langle (\delta n_u + \delta n_d)^2 \right\rangle \tag{25}$$

and

$$\chi_{NS} = V_{qg}\beta \left\langle (\delta n_u - \delta n_d)^2 \right\rangle \tag{26}$$

and found that at high temperature both are very close to the non-interacting thermal gas limit

$$\chi_S \approx \chi_{NS} \approx 2T^2 \tag{27}$$

From this result, one can first of all infer that u and d quark densities are uncorrelated

$$\langle \delta N_u \delta N_d \rangle \approx 0 \tag{28}$$

and

$$\left\langle \delta N_u^2 \right\rangle \approx \left\langle \delta N_d^2 \right\rangle \ . \tag{29}$$

The results of Ref. [9] then implies that the charge fluctuation at high temperature follows that of the thermal fermion gas

$$\left\langle \delta Q^2 \right\rangle = \frac{4}{9} \left\langle \delta N_u^2 \right\rangle + \frac{1}{9} \left\langle \delta N_d^2 \right\rangle \tag{30}$$

or

$$T\chi_q = \frac{5}{9} T^3 \ . \tag{31}$$

For the charged multiplicity, we assume that the relations Eqs. (20 – 21) still hold. Equating the entropy of the final pions with that of the primordial quarks and gluons, one obtains

$$\langle N_{\rm ch} \rangle = \frac{1}{5.4} V_{qg} \langle \sigma_{qg} \rangle \ . \tag{32}$$

Ref. [9] reports that the gluon entropy density σ_g is almost the same as the non-interacting thermal bosons, but the quark entropy density σ_{u+d} is about one half of that of the non-interacting thermal fermions. Hence, the total entropy from the lattice calculation is

$$\begin{aligned} \langle \sigma_{qg} \rangle &= \langle \sigma_g \rangle + \langle \sigma_{u+d} \rangle \\ &= 16 \times 3.6 \times \langle f_g \rangle + 24 \times 4.2 \times \alpha \langle f_q \rangle \\ &\approx 12\, T^3 \end{aligned} \tag{33}$$

where $\alpha \approx 1/2$ and $\langle f_{q,g} \rangle$ are the average density per degree of freedom

The lattice result is then

$$\frac{\left\langle \delta Q^2 \right\rangle}{\langle N_{\rm ch} \rangle} = 5.4 \frac{T\chi_q}{\sigma_{qg}} \approx \frac{1}{4} \tag{34}$$

Equivalently,

$$\langle N_{\rm ch} \rangle \left\langle \delta R^2 \right\rangle \approx 1 \tag{35}$$

which is still 4 times smaller than the pion gas result. This is the main result of this paper. The difference between the pion gas result and the QGP result is distinct enough one should easily see it in the first few days of data collecting at RHIC. This is also one of the rare instance where what is calculated on lattice can be directly observed in an experiment.

The picture obtained above holds if the following two conditions are met: (i) The detected phase space is a small sub-system of the whole. (ii) We do not lose original quarks and gluons during or after the hadronization. In terms of the rapidity intervals, the same conditions can be characterized as

$$y_{\rm total} \gg y_{\rm accept} \gg 1 \ . \tag{36}$$

Here, $y_{\rm total}$ is the rapidity range allowed by the energetics of the collisions and $y_{\rm accept}$ is the acceptance interval of a given detector. The first of these conditions is needed to ensure that the rest of the system acts as a reservoir and the second condition ensures that the charge diffusion in the rapidity space during and after hadronization is negligible. In real life, of course, Eq. (36) is satisfied in varying degrees. For instance, the STAR at RHIC has $y_{\rm total} \approx 10$, $y_{\rm accept} \approx 3$ and hence corrections should be taken into account. We would like now to discuss Caveats and corrections due to these and other effects.

Resonance Contributions : As explained in a previous paper [5], neutral resonances introduce positive correlations between N_+ and N_- and hence lower the value of $\langle N_{ch} \rangle \left\langle \delta R^2 \right\rangle$. In a thermal scenario studied in the same paper, we found that the resonances reduce the fluctuation by about 30 %. This implies that realistic hadron gas gives

$$\langle N_{ch} \rangle \left\langle \delta R^2 \right\rangle \approx 3 \ . \tag{37}$$

This is still a factor of 3 bigger than the lattice result.

Mixtures : If the system is a mixture of a QGP and a hadron gas, the signal should depend on the fractions. To a first approximation, it should be a linear combination of Eqs. (37) and (35)

$$\langle N_{\rm ch} \rangle \left\langle \delta R^2 \right\rangle = 3(1-f) + f$$

where f is the QGP fraction. Even if $f \sim 0.5$, the signal can be still visible.

Rapidity Correlations : The charged particles in pp collisions are said to be strongly correlated in the rapidity space [11] with the correlation length of $\Delta y \approx 1$. If such a strong correlation holds in AA collisions as well, it will further reduce the value of $\langle N_{\rm ch} \rangle \left\langle \delta R^2 \right\rangle$ for the hadronic gas result. Fortunately, even in pp the strong correlation is visible only in the 'connected' correlation function

$$C(y_+, y_-) = \rho_2(y_+, y_-) - \rho_1(y_+)\rho_1(y_-)$$

where ρ_n is the n-particle density function. The correlation in $\rho_2(y_+, y_-)$ itself which is of relevance to us is not so strong. This fact can be readily established from the definitions and data in Ref. [11].

Hadronization : The signal should survive hadronization since both the charge and the entropy are conserved. Once it is fixed in an event, it should not matter what the final states are in that given event.

Finite Acceptance Correction : The finite size of the acceptance window introduces a factor of $(1-p)$ corrections where p is the fraction of the total multiplicity inside the acceptance window. This is easy to understand. If the detector sees a 100 % of all charged particles ($p = 1$), the fluctuations should shrink down to zero due to the global charge conservation. Fortunately, this is a common factor that applies to both Eqs. (37) and (35) so that the ratio stays the same.

Effects of Rescattering : The partons as well as hadrons are subject to rescattering during the course of a heavy ion collision. This in principle may affect the above charge fluctuations by diffusing the charge in rapidity However in the limit $y_{\rm total} \gg y_{\rm accept} \gg 1$ these effects should be very small since they scale as the surface to volume ratio in rapidity space. To estimate the effect in the hadronic phase, we performed a simple Monte Carlo calculation where a Gaussian noise with $\sigma = 0.5$ are added to each particle's rapidity originating from a highly correlated source. Doing so increases the value of $\langle N_{\rm ch}\rangle \langle \delta R^2\rangle$ up to 40% assuming the STAR acceptance at RHIC.

Strangeness : Adding Kaons to a pion gas will not change the value of $\langle \delta Q^2\rangle / \langle N_{\rm ch}\rangle$ because their contribution adds exactly the same amount to both numerator (c.f. Eq.(11)) and the denominator. For a quark gluon plasma, the lattice calculation [9] suggests that at high temperature, the strangeness entropy is about 40 % of the $u + d$ entropy. Taking this at a face value changes the result (35) by less than 10 %.

In conclusion, we showed in this paper that 'Day-1' detection of QGP formation is quite possible through the measurement of h^+/h^- fluctuation. This measurement should be very feasible for STAR. We also emphasize that this is a direct confirmation of the lattice QCD results. What we considered here is the simplest ratio out of many possible ones that can behave quite differently in the presence of a QGP. For instance, the strangeness anti-strangeness ratio fluctuations can provide us with a valuable handle on the strangeness distributions with or without the formation of a QGP. These and related issues are under active investigation and will be reported elsewhere [7].

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Note added: After finshing this work, we received a preprint by Asakawa, Heinz and Müller [12] which addresses similar issues.

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